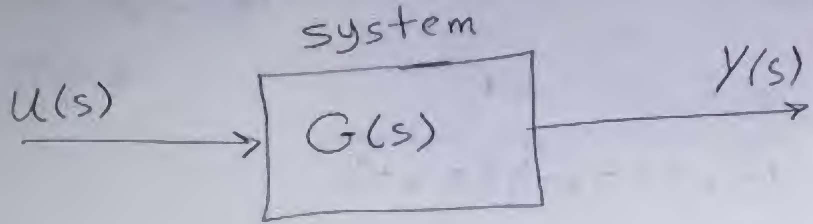
$\boxed{19}$ محاضره نظم وإشارات

→ state-space model



$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$

Given :- $G(s) = \dfrac{Y(s)}{U(s)}$

(1) Controllable Canonical from (CCF)

(2) observable " " (OCF)

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(3) Paraller [diagonal] Form

Example $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{2s^2 + 5s + 1}{s^3 + 6s^2 + 11s + 6}$

different from each other

$\boxed{\text{Sol}}$

ch. eqn → $s^3 + 6s^2 + 11s + 6$ → roots = Poles

(1) real and distinct

(2) real & equal.

(3) Complex Conjugate.

ـــ ممكن يجيب في المسائل
معادلة تفاضلية ويطلب حلها بالطريقة
دي هنحسب (Laplace) لها
والحل يكون كالعادي.

Poles

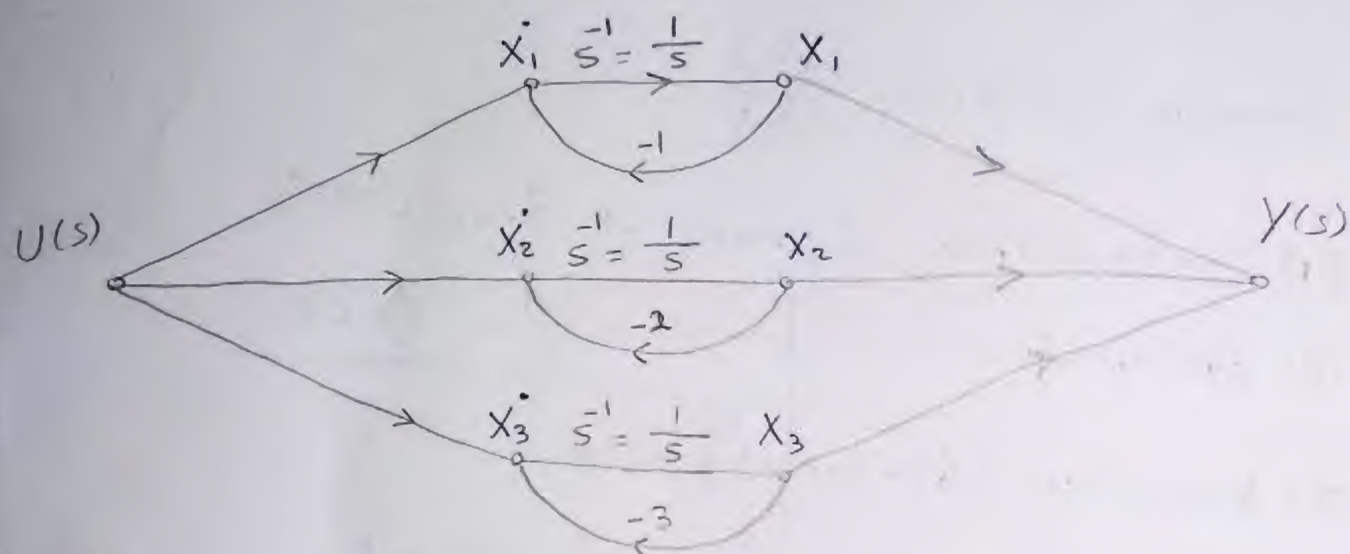
$\lambda_1 = -1$

$\lambda_2 = -2$

$\lambda = -3$

$$\therefore G(s) = \frac{2s^2 + 5s + 1}{(s+1)(s+2)(s+3)}$$

$$= \boxed{\frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}}$$



diagonal form

Postmultiplication

ـــ يجعل ال (branches) الخارجة من $U(s) = 1$ ، الداخلة لـ $Y(s) = K_1, K_2, K_3$.

Premultiplication

ـــ يجعل ال (branches) الخارجة من $U(s) = K_1, K_2, K_3$ و الداخلة لـ $Y(s) = 1, 1, 1$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \underline{\underline{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underline{\underline{\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}}} u(t)$$

$$y(t) = \underline{\underline{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \cancel{D}\, u(t) \quad D \to 0$$

ثم ننظر إلى (signal flow graph) وتملئ الأماكن في القانون السابق (القيم المضافة بالرموز هي الحل)

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(4) Cascaded [series] form

Given: $$G(s) = \frac{Y(s)}{U(s)} = \frac{10\,(s+3)(s+8)}{(s+4)(s+5)(s+6)}$$

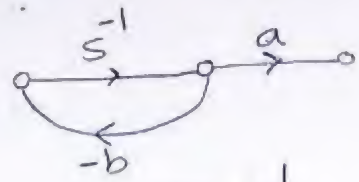
$$\frac{Y(s)}{R(s)} = \frac{10s^2 + 110s + 240}{s^3 + 15s^2 + 74s + 120}$$ ← ده أساس المعادلة

$$\frac{Y(s)}{U(s)} = \left(\frac{10}{s+4}\right) \cdot \left(\frac{s+3}{s+5}\right) \left(\frac{s+8}{s+6}\right)$$

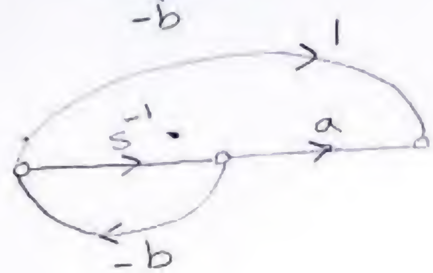
[3]

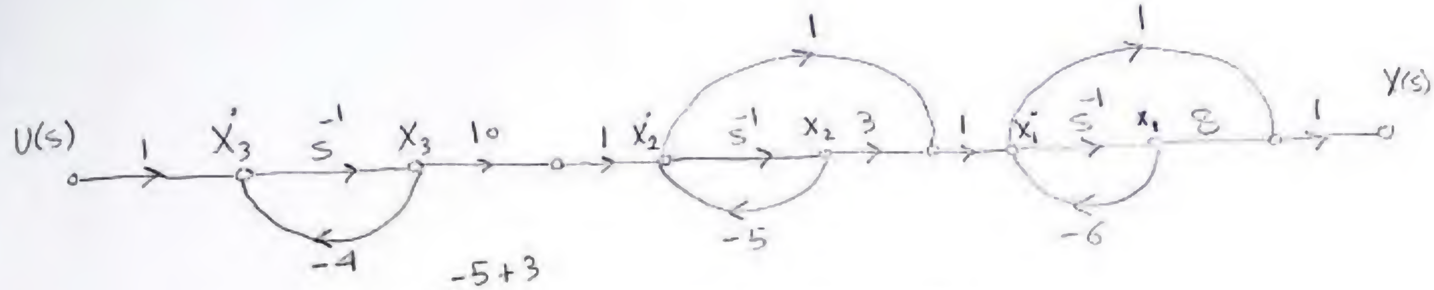
Note

$$\frac{a}{s+b} \longrightarrow$$



$$\frac{s+a}{s+b} \longrightarrow$$





$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -6 & \overset{-5+3}{\underline{\underline{-2}}} & 10 \\ 0 & -5 & 10 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} \overset{8+(-6)}{2} & \overset{3+(-5)}{-2} & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0\,u(t)$$

← القيمة (-2) ← لما مسارين 3 ، -5 نجمعهم معاً

$\boxed{\text{السؤال هيجي كده}}$ $\boxed{4}$ Lec 19

→ obtain two different models for state space representation.

→ time response of the system

Given: $G(s) = \frac{Y(s)}{U(s)} \Rightarrow Y(s) = G(s) \cdot U(s)$

$= \left[G(s) \cdot \frac{1}{s} \right]$

in unit step

$u(t) = 1$

or

Given

$$\boxed{\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t) + D\,u(t) \end{aligned}}$$ → *

→ first-order D.E.

→ Find unit-step response

Note $\boxed{I}$ $G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$

نحسب G(s) من القانون ده ومنه نحسب (response) Y(s)

٢- الطريقة الثانية إن نتعامل مع القانون * .

$$x(t) = \phi(t)\,x(0) + \int_0^t \phi(t-\tau)\,B\,u(\tau)\,d\tau$$

where $u(\tau) = u(t)\Big|_{t=\tau}$

$u(t) = 1 \Rightarrow u(\tau) = 1$

$$X(0) = \begin{bmatrix} X_1(0) \\ X_2(0) \\ \vdots \\ X_n(0) \end{bmatrix} = \text{initial state vector} = x(t)\Big|_{t=0}$$

$$\phi(t) = \begin{bmatrix} & \end{bmatrix}_{n \times r} = \text{state transition matrix}$$

$$\phi(t) = \mathcal{L}^{-1} (SI - A)^{-1}_{2 \times 2}$$

<u>Given</u> $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$$Y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(t)$$

<u>Find</u>: ① <u>unit-step response</u> for

$$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$y(t) = ??$ when

$u(t) = 1 = \text{unit-step}$

<u>Sol</u>

$$\phi(t) = \mathcal{L}^{-1} (SI - A)^{-1}$$

عناصر القطر الرئيسي $\dot{x}(t)$

نضربها في s

$$SI - A = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$(sI-A)^{-1} = \begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \\[2ex] \dfrac{2}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \end{bmatrix}$$

المقام الموجود في المصفوفة هو نفس محدد المصفوفة
$(sI-A)$

$$s(s+3) - (-1 \times 2)$$

$$\phi(t) = \mathcal{L}^{-1} \begin{bmatrix} \dfrac{2}{s+1} + \dfrac{-1}{s+2} & \dfrac{1}{s+1} + \dfrac{-1}{s+2} \\[2ex] \dfrac{-2}{s+1} + \dfrac{2}{s+2} & \dfrac{-1}{s+1} + \dfrac{2}{s+2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$\phi(0) = \phi(t)\big|_{t=0}$ ← عليها (check) لازم نعمل ←

$$\phi(0) = \phi(t)\Big|_{t=0} = I$$

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$$x(t) = \phi(t) \cdot x(0) + \underbrace{\int_0^t \phi(t-\tau) B\, u(\tau) \cdot d\tau}_{I_1}$$

$$I_1 = \int_0^t \begin{bmatrix} 2e^{-(t-\tau)} - e^{-2(t-\tau)} & e^{-(t-\tau)} - e^{-2(t-\tau)} \\ -2e^{-(t-\tau)} + 2e^{-2(t-\tau)} & -e^{-(t-\tau)} + 2e^{-2(t-\tau)} \end{bmatrix}$$

$$* \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau$$

$$= \int_0^t \begin{bmatrix} e^{-(t-\tau)} - e^{-2(t-\tau)} \\ -e^{-(t-\tau)} + 2e^{-2(t-\tau)} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} e^{-(t-\tau)} - \frac{1}{2} e^{-2(t-\tau)} \\ -e^{-(t-\tau)} + e^{-2(t-\tau)} \end{bmatrix} \Bigg|_0^t = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} - \begin{bmatrix} e^{-t} - \frac{1}{2} e^{-2t} \\ -e^{-t} + e^{-2t} \end{bmatrix}$$

$$I_1 = \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}_{2 \times 1}$$

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$$x(t) = \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2}}_{\phi(t)} \begin{bmatrix} 1 \\ -1 \end{bmatrix}_{2\times 1} + \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

$$\therefore x_1(t) = \frac{1}{2} + \frac{1}{2} e^{-2t}$$

$$x_2(t) = -e^{-2t}$$

نفرض $t = 0$ حتى تتأكد

انها تساوي $\begin{bmatrix} 1 \\ -1 \end{bmatrix} = x(0)$

$$y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} + \frac{1}{2} e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

$$y(t) = \frac{1}{2} + \frac{3}{2} e^{-2t} \longrightarrow \text{unit-step response}$$

→ stability of system
- → stable
- → unstable
- → critically stable

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→ check the system stability

لو ظهرت حتى يتم حلها نحسب ch.eqn للنظام.

حل على المثال السابق.

→ another solution:-

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$$

A → system matrix

ch-eqn ⇒ $|sI - A| = 0$

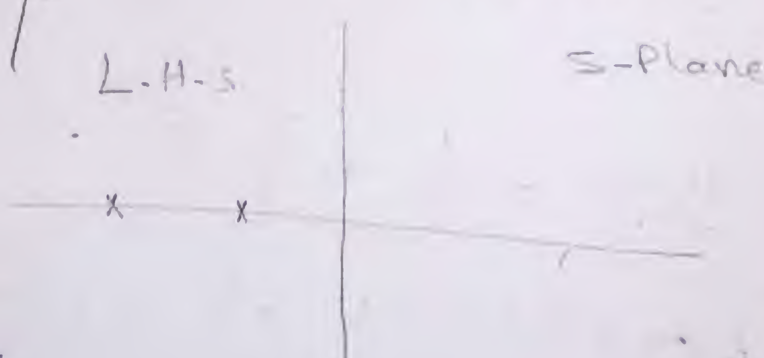
$$|sI - A| = \begin{vmatrix} s & -1 \\ -2 & s+3 \end{vmatrix} = \boxed{s^2 + 3s + 2} = 0$$

→ Poles ≡ eigen values

$\lambda_1 = -1$, $\lambda_2 = -2$

تعني أن النظام

(stable)



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